May 12, 2014
Time : 90 minutes
Spring 2013-14

MATHEMATICS 218
Final Examination

NAME -------------------
ID# ------------------

Circle your section number :

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1	2	3	4	5	6	7	8	9	10	11	12
9 M	2 F	8 M	1 W	2 F	1 M	3:30 T	5 T	12:30 T	1 F	11 M	11 F

PROBLEM GRADE

PART I

1 ------- /16

2 ------- / 18

3 ------- / 10

4. ------- / 12

5. ------- / 7

PART II

6	**7**	**8**	**9**	**10**	**11**	**12**
a	a	a	a	a	a	a
b	b	b	b	b	b	b
c	c	c	c	c	c	c
d	d	d	d	d	d	d

6-12 ------- / 21

PART III

a	**b**	**c**	**d**	**e**	**f**	**g**	**h**

13 ------- / 16

TOTAL ------- /100

PART I. **Answer each of the following problems in the space provided for each problem (Problem 1 to Problem 5).**

1. Let $A=\begin{pmatrix} 5 & 1 & 0 \\ 1 & 5 & 0 \\ 0 & 0 & 6 \end{pmatrix}$

(a) Find the eigenvalues and a basis for each eigenspace of A.

[10 points]

1(b) Show that A is diagonalizable. Find an invertible matrix P and a diagonal matrix D such that $P^{-1}AP=D$. (Do not verify)

[6 points]

2. Let $T : P_2 \rightarrow P_2$ be the linear transformation defined by $T(p(x)) = xp'(x)$.

(a) Find the matrix A=$[T]_{\beta}$ of T relative to the standard ordered basis $\beta = \{1, x, x^2\}$ of P_2 .

[6 points]

(b) Find the matrix B=$[T]_{\beta'}$ of T relative to the ordered basis $\beta' = \{1+x^2, 2x, 1\}$ of P_2.

[6 points]

(c) Find the transition matrix P=$[I]_{\beta'}^{\beta}$ from β' to β such that B=P^{-1}AP (Do not verify).

[6 points]

3. Prove that an orthogonal set of nonzero vectors in an inner product space V is linearly independent.

[10 points]

4. Let T: $R^3 \rightarrow R^3$ be the linear transformation defined by

$$T\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} a+b+c \\ 0 \\ 0 \end{pmatrix}.$$

Find a basis for the null space N(T) and use the Gram-Schmidt process to construct an orthonormal basis for N(T), with the usual dot product.

[12 points]

5. Let T: $R^3 \rightarrow R^4$ and S: $R^4 \rightarrow R^2$ be two linear transformations such that the composition $S \circ T = 0$. Show that if S is onto then T cannot be one-to-one.

[7 points]

<u>PART II.</u> Circle the correct answer for each of the following problems (Problem 6 to Problem 12). <u>IN THE TABLE IN THE FRONT PAGE</u> [3 points for each correct answer]

6. Let $T:\mathbf{R}^3 \to \mathbf{R}^3$ be the linear map defined by

$$T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3x+2y \\ -x \\ 0 \end{pmatrix}.$$

Then dim (Range T) is :

a. 3
b. 2
c. 1
d. none of the above.

[3 points]

7. Let S be the subspace defined by
S={M is a 3x3 skew-symmetric matrix with the sum of the entries of each row is zero}.
Then dim S=

a. 1
b. 2
c. 3
d. none of the above.

[3 points]

8. If T: $\mathbb{R}^2 \to \mathbb{R}$ is a linear transformation such that T(**v**)≠0 for some **v** in R^2, then

a. T is one-to-one.
b. T is onto.
c. Dim (Nullspace T)=0.
d. none of the above.

[3 points]

9. Let A= $\begin{pmatrix} 2 & 2 \\ 1 & 2 \\ 1 & 1 \end{pmatrix}$ and b= $\begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix}$. Then the least squares solution to $Ax = b$ is:

a. $\hat{x} = \begin{pmatrix} 3/5 \\ 6/5 \end{pmatrix}$

b. $\hat{x} = \begin{pmatrix} -6/5 \\ 3/5 \end{pmatrix}$

c. $\hat{x} = \begin{pmatrix} -3/5 \\ 6/5 \end{pmatrix}$

d. None of the above

[3 points]

10. Let U be the subspace of R^3 defined by

$$U = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in R^3 \mid x - 2y + 3z = 0 \right\}.$$ Then $\dim U^{\perp} =$

a. 3
b. 1
c. 2
d. none of the above

[3 points]

11. Let A= $\begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 8 \\ 0 & 1 & 3 \end{pmatrix}$. Then:

a. A is diagonalizable.
b. A is not invertible.
c. The eigenspace corresponding to the eigenvalue λ=1 has dimension 2.
d. None of the above.

[3 points]

12. Let T: V$\rightarrow$V be a linear transformation with dim V= n such that T is <u>onto</u>. Which one of the following statements is <u>FALSE:</u>

a. T is an isomorphism
b. If { $v_1, v_2, \ldots, v_n$} is linearly independent in V, then {$T(v_1), T(v_2), \ldots, T(v_n)$} is linearly independent in V.
c. dim(RangeT)=n
d. none of the above.

[3 points]

PART III. Answer **TRUE** or **FALSE** only **IN THE TABLE IN THE FRONT PAGE** (**2 points for each correct answer)**

a. ----- If A is a 3×3 matrix such that $A^2=0$, then rank $A \neq 3$.

b. ----- Let V be a finite dimensional vector space and let W be a subspace of V. If dim W= dim V, then W=V.

c. ----- If A is a 2×5 matrix, then dim (Column space of A) ≤ 2

d. ------ Let V be a finite dimensional inner product space and let W be a subspace of V. Then the orthogonal complement of $W^{\perp}$ is equal to W.

e. ------- Let V be a finite-dimensional vector space and let T: V→V be a linear transformation. If T is one-to-one, then T is onto.

f. ----- The matrices $A=\begin{pmatrix} 1 & -2 \\ 3 & 4 \end{pmatrix}$ and $B=\begin{pmatrix} -2 & 1 \\ 4 & 3 \end{pmatrix}$ are similar.

g. ---- Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation such that

$$T\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \text{ and } T\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}.$$

Then $T\begin{pmatrix} 5 \\ -1 \end{pmatrix} = \begin{pmatrix} 4 \\ 7 \\ -2 \end{pmatrix}$.

h. ----- The set of all 2×2 noninvertible matrices is a subspace of $M_{2\times 2}$.

[16 points].

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